Z5 – Zeeman effect in the Hg spectrum measured with the Fabry-Pérot interferometer

Physics Laboratory II Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University

The aim of the experiment is to gain experience in the field of an optical high resolution spectroscopy. Students investigate the Zeeman splitting of the mercury line ($\lambda = 546.1$ nm) in the magnetic field of up to 1.5 T. High spectral resolution is provided by the Fabry-Pérot interferometer with dielectric mirrors. The distance between the mirrors is $d = 3.54$ mm and the spectral tuning is accomplished by changing the pressure of the CO_2 gas in the interferometer chamber. When using a polarizer, both the π and σ components of the Zeeman spectrum may be observed, along the direction perpendicular to the direction of the magnetic field. The observation of the $\sigma^{\pm}$ components along the magnetic field direction is also possible. On the basis of the registered interferograms the students calculate the Zeeman splitting of the Hg line as well as the magnetic field induction.

Preparatory questions

The laboratory starts with an oral test. First week: questions 1. and 2., exercises 1. and 2. Second week: question 3. and exercise 3. and 4. in the extended version.

First week:

1. Fabry-Pérot interferometer ([1, 2], [3] experiment 11 and Appendices I and II, [5] page 216-224, **[6]**):
   - 1.1. basic information about interference, coherence, multiple beam interference,
   - 1.2. construction, principle of operation and applications of the Fabry-Pérot interferometer,
   - 1.3. Fabry-Pérot interferometer parameters:
     - a) theoretical resolving power R ([1] page 414, **[3] page 94, 119**):
       - definition,
       - comparison of the Rayleigh criterion for a diffraction grating and the F-P interferometer,
     - b) instrumental linewidth $\Delta\tilde{\nu}_{1/2}$ ([1] page 410-414, [3] page 119):
       - Airy function,
       - definition and expression for the instrumental linewidth (expressed as a fraction of the order $\gamma_{1/2}$ and in wavenumbers), $\Delta\tilde{\nu}_{1/2}$ [cm^{-1}],
       - dielectric mirrors as a method to reduce the instrumental linewidth, $\Delta\tilde{\nu}_{1/2}$; basic properties of the dielectric mirrors, comparison to metallic mirrors ([3] page 129-131),
     - c) free spectral range FSR $\Delta\tilde{\nu}_{dysp}$ (**[1] page 412-414**, [3] page 119):
       - definition and physical meaning,
       - expression for FSR with derivation (in wavenumbers $\Delta\tilde{\nu}_{dysp}$ and in wavelengths $\Delta\tilde{\lambda}_{dysp}$,
       - relation between the resolving power R and FSR and finesse,
     - d) finesse, types of finesse,
     - e) angular dispersion D_ϕ [1] page 413, (**[3] page 118**),
   - 1.4. the effect influencing the spectral linewidth:
     - 1.4.1. instrumental effects:
       - a) instrumental linewidth $\Delta\tilde{\nu}_{1/2}$ (section 1.3.b above),
       - b) temperature t inhomogeneity in the interferometer region, $\Delta\tilde{\nu}_t$ ([2] page 144, **[3] page 442**),
       - c) pressure p change in the interferometer region, ([2] page 144, **[3] page 442**),
       - d) imperfections of the mirror surfaces (flatness and roughness), $\Delta\tilde{\nu}_{pow}$ ([2] page 165),
       - e) imperfections in the interferometer adjustment – wedged mirrors, $\Delta\tilde{\nu}_j$ (see section 1.5 below),
       - f) spectral width of the rectangular profile $\Delta\nu_{apert}$ connected with a finite diameter D of the circular aperture placed in front of the photomultiplier (see exercise 1 (e) below),
     - 1.4.2. effects connected with the light source (e.g. [3, 4], [5] page 86):
       - a) Doppler effect $\Delta\nu_D$ ([3] page 267-269), (see exercise 1 a),
       - b) natural width $\Delta\nu_{rad}$ ([3] page 264),
       - c) pressure and Stark broadening (e.g. [3] page 277),
       - d) broadening caused by the magnetic field inhomogeneity,
   - 1.5. methods of interferometer adjustment ([2] page 138-144, [3] page 437-443):
     - a) initial adjustment using only the reflections of light,
     - b) fringes of equal thickness (Fizeau fringes),
     - c) fringes of equal inclination (Haidinger rings),

1.6. ghost fringes in the interference pattern (how do they appear, how to eliminate them, [2] page 145),

2. How does the photomultiplier work? (e.g. [3] page 31),

Second week:

3. Zeeman effect (e.g. [3, 4]) – basic description, selection rules, polarization of the Zeeman components.

Computational assignments

First week:

1. Calculate in [cm^{-1}] the contributions to the experimental effective spectral width of the interference fringe $\Delta\tilde{\nu}_{1/2}^{exp}$:
   (a) the Doppler effect, assuming that the temperature of the lamp is between 100–150°C; $\Delta\tilde{\nu}_D = 0,716 \cdot 10^{-6}\tilde{\nu}\sqrt{\frac{T}{M}}$ [cm^{-1}], where T –temperature in [K], M – mass number and $\tilde{\nu}$ – wavenumber of the Hg spectral line $\lambda = 546.1$ nm,
   (b) instrumental width of the interference fringe $\Delta\tilde{\nu}_{1/2}$; use the parameters of the interferometer given in *Apparatus and materials* section,
   (c) the isotope effect and the hyperfine structure. The spectral lamp contains a natural mixture of Hg isotopes, so the spectral line has a rich isotope and hyperfine spectral structure. Using the data from [3], compare this structure with the FSR $\Delta\tilde{\nu}_{dysp}$ of the interferometer. Estimate the expected broadening of the spectral line $\Delta\tilde{\nu}_{izotop}$.
   (d) the broadening of the spectral line caused by imperfections of the mirror surface ([2], page 165). Calculate $\Delta\tilde{\nu}_{pow}$ assuming that the flatness of both mirrors is $\lambda/200$.
   (e) the width of the rectangular line profile $\Delta\tilde{\nu}_{apert}$ connected with a finite diameter D of the entrance aperture placed in front of the photomultiplier. The diameter equals $D_1 = 2.20$ mm, $D_2 = 1.55$ mm or $D_3 = 0.70$ mm. Assume that the focal length of the S3 lens is $f_3 = 45$ cm.

   Hint: $\Delta\tilde{\nu}_{apert}$ may be considered as a difference in the wavenumbers of two infinitely narrow spectral lines $\tilde{\nu}_a$ and $\tilde{\nu}_b$, for which the interference maxima of the same order appear in the middle of the interference pattern for one line and at the border of the diaphragm D for the second line ([3] page 447, equation (14) substituting $d\lambda/\lambda = d\tilde{\nu}/\tilde{\nu}$ i.e. $\Delta\tilde{\nu}_{apert} = \tilde{\nu}\{(D_{a,m}^2 - D_{b,m}^2)/(8f^2)\}$.

2. Taking into account the results of the previous exercises estimate (in cm^{-1}) the effective spectral width of the interference fringe $\Delta\tilde{\nu}_{1/2}^{exp}$. For simplicity, calculate it as an algebraic sum of the contributions $\Delta\tilde{\nu}_D$, $\Delta\tilde{\nu}_{1/2}$, $\Delta\tilde{\nu}_{izotop}$, $\Delta\tilde{\nu}_{pow}$ and $\Delta\tilde{\nu}_{apert}$.

Second week:

3. Exercises concerning the Zeeman splitting of the Hg line $\lambda = 546.1$ nm (Fig. 3 and 4 in [7])
   (a) On the basis of Fig. 3 in [7] find the electron configuration of the upper and lower energy level of the Hg line $\lambda = 546.1$ nm. Read the notation of the electron levels in the L-S coupling.
   (b) On the basis of Fig. 4 in [7] analyze the scheme of the electron levels in the Zeeman splitting of the Hg line in the magnetic field. Find the polarization of the Zeeman components for the observation perpendicular and parallel to the direction of the magnetic field B ([3] page 318).
   (c) Calculate the spectral distance (in cm^{-1}) between Zeeman components for the magnetic field of 1.5 T – see Fig. 4 in [7] and page 319 in [3], Fig. 14. The energy change ΔE of a given level is given by:
   $$\Delta E = \mu_0 M_J g B,$$
   where: μ_0 – Bohr magneton equal $e\hbar/(2m_e)$ (m_e – electro mass), M_J – magnetic quantum number, $M_J = -J \ldots J$, $M \in \mathbb{Z}$, g – Landé factor for the level described by the quantum numbers L, S, J:
   $$g = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}.$$
   The above equation for ΔE is approximately correct for our experimental conditions. Remember however that it is not correct in a general case.
   (d) Using the equations shown in Table 1 in [7], calculate the relative intensities of the Zeeman components of the Hg line $\lambda = 546.1$ nm, for the observation parallel and perpendicular to the direction of the magnetic field B.

4. How to distinguish σ^+ and σ^- components using a polarizer and a quarter waveplate?

Apparatus and materials

1. Fabry-Pérot interferometer with a vacuum setup used to scan the interferometer by changing the pressure of CO_2 gas. The distance between interferometer mirrors is $d = (3.540 \pm 0.005)$ mm. The reflection coefficients of the dielectric mirrors is $R = (0.94 \pm 0.01)$ for wavelengths in the range of 500 to 560 nm.

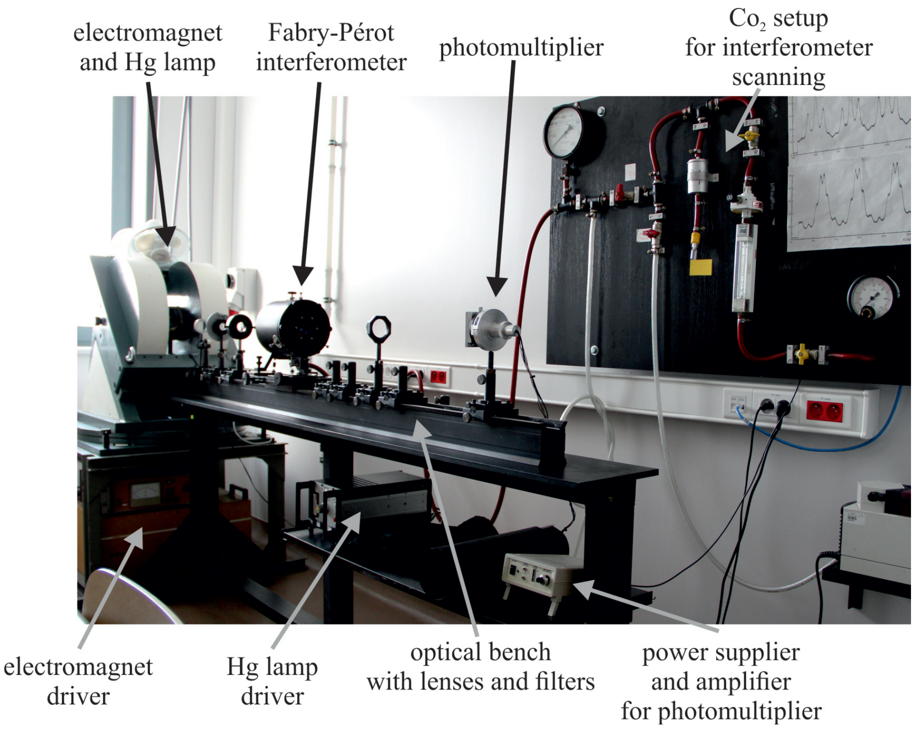

Figure 1: Experimental setup

2. Hg spectral lamp with a driver.

3. Electromagnet producing magnetic field up to about 1.5 T.

4. Optical setup consisting of lenses, iris diaphragm, polarizer, quarter waveplate (for the observation along the magnetic field direction), diffusive plate, optical band-pass filter and screens with small diaphragms of $D_1 = 2.20$ mm, $D_2 = 1.55$ mm and $D_3 = 0.70$ mm in diameter.

5. Hamamatsu Photonics H10722 Photomultiplier with driver, regulated amplifier and analog to digital converter. The signal from the converter is sent to a computer via USB link and registered with a RUM software.

Experiment

1. Preparation of the optical setup. Adjustment of the Fabry-Pérot interferometer using method of fringes of equal inclination.

2. Analysis of the interferogram for the zero magnetic field.

3. Calculation of the Zeeman splitting for π components for a few values of the magnetic field (up to 1.5 T). Interpretation and comparison of the interferograms for π and σ components. Calculation of the magnetic field induction on the basis of the interferograms for the π components.

4. Extended version: observation of the Zeeman splitting in the direction parallel to the direction of the magnetic field. σ^+ and σ^- components may be distinguished using a polarizer and a quarter waveplate.

Data analysis

For all the registered interferograms, the linearity of the interferometer scan should be performed. Next, knowing the interferometer FSR (on the basis of the distance between mirrors), the distance between the π components of the Zeeman splitting should be calculated. Important: the calculations should be done separately for each interferogram, since the speed of scanning is slightly different for different measurements.

The Zeeman splittings in all orders in a given interferogram (i.e. for a given electromagnet current) should be averaged. Knowing the Zeeman splitting, the magnetic field induction should be calculated.

The uncertainties of the measured Zeeman splitting and magnetic field induction should be estimated on the basis of: a) the uncertainty of the interferometer FSR value, b) the uncertainty of the positions of the Zeeman components in the interferogram, c) dispersion of the Zeeman splitting values for all the orders in a given interferogram.

Safety rules

1. The electromagnet is water-cooled. After the experiment the water valve should be closed.

2. The magnetic field is dangerous for people having a pacemaker.
3. Avoid bringing magnetic sensitive devices close to the electromagnet (like watches, bank cards).
4. Do not touch the spectral lamp nor its connections (the lamp gets hot during operation).
5. Avoid looking directly at the lamp when it operates in arc discharge mode. The Hg lamp is a source of UV radiation.
6. Switching on any electrical devices and handling of the vacuum system may be done only with the consent of the supervisor.

References

[1] K. Meissner, *Interference Spectroscopy*, part I, JOSA **31**, 405-426 (1941).

[2] S. Tolansky, *High Resolution Spectroscopy*, chapter 9.

[3] R. I. Sołouchin (editor), *Optyka i fizyka atomowa – ćwiczenia laboratoryjne (Optics and atomic physics)*, PWN, 1982.

[4] Any textbook: *Introduction to atomic spectroscopy.*

[5] A. N. Matveev, *Optics*, Mir Publishers 1988.

[6] Geometrical and wave optics at the level of the university introductory course.

[7] Manual of the Z5 experiment.