Solid-state Physics

Conduction phenomena
Hall Effect

Physics Leaflets

P7.2.1.4

Determining the density and mobility of charge carriers in p-germanium

Objects of the experiment

- Measuring of the Hall voltage as function of the current at a constant magnetic field: determination of the density and mobility of charge carriers.
- Measuring of the Hall voltage for as function of the magnetic field at a constant current:: determination of the Hall coefficient.
- Measuring of the Hall voltage as function of temperature: investigation of the transition from extrinsic to intrinsic conductivity.

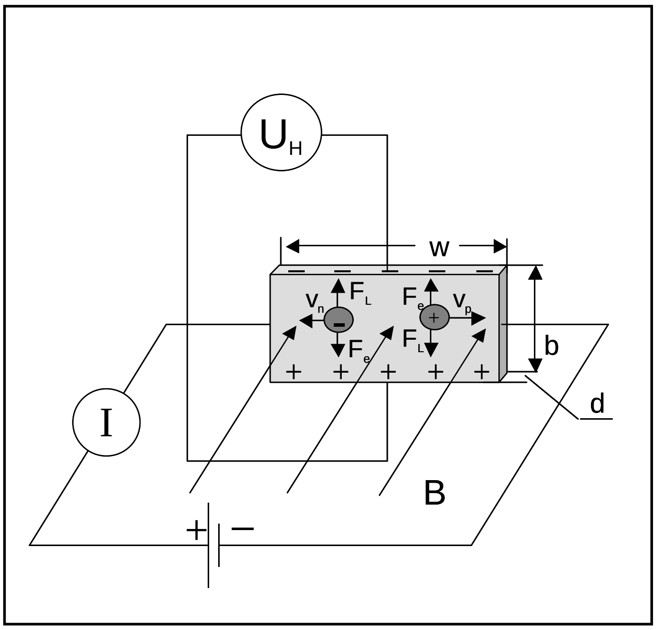

Fig. 1: Hall effect in a rectangular sample of thickness d, height b and length w: At equilibrium conditions the Lorentz force F_L acting on the moving charge carriers is balanced by the electrical force F_e which is due to the electric field of the Hall effect.

Principles

The Hall effect is an important experimental method of investigation to determine the microscopic parameters of the charge transport in metals or doped semiconductors.

To investigate the Hall effect in this experiment a rectangular strip of p-doped germanium is placed in a uniform magnetic field B according Fig. 1. If a current I flows through the rectangular shaped sample an electrical voltage (Hall voltage) is set up perpendicular to the magnetic field B and the current I due to the Hall effect:

$$U_H = R_H \cdot \frac{I \cdot B}{d} \qquad \text{(I)}$$

R_H is the Hall coefficient which depends on the material and the temperature. At equilibrium conditions (Fig. 1) for weak magnetic fields the Hall coefficient R_H can be expressed as function of the charge density (carrier concentration) and the mobility of electrons and holes:

$$R_H = \frac{1}{e_0} \cdot \frac{p \cdot \mu_p^2 - n \cdot \mu_n^2}{(p \cdot \mu_p + n \cdot \mu_n)^2} \qquad \text{(II)}$$

$e_0 = 1.602 \cdot 10^{-19}$ As (elementary charge)
$p = p_E + p_S$ (total density of holes)
p_E: density of holes (intrinsic conduction)
p_S: density of holes (hole conduction due to p-doping)
$n = n_E$:density of electrons (intrinsic conduction)
μ_p:mobility of holes
μ_n mobility of electrons

From equation (II) follows: The polarity of predominant charge carriers can be determined from the Hall coefficient R_H if the directions of the current I and magnetic field B are known. The thinner the conducting strip the higher the Hall voltage.

The doping of group III elements like e.g. B, Al, In or Ga into the crystal lattice of germanium creates positive charged holes in the valence band (Fig. 2).

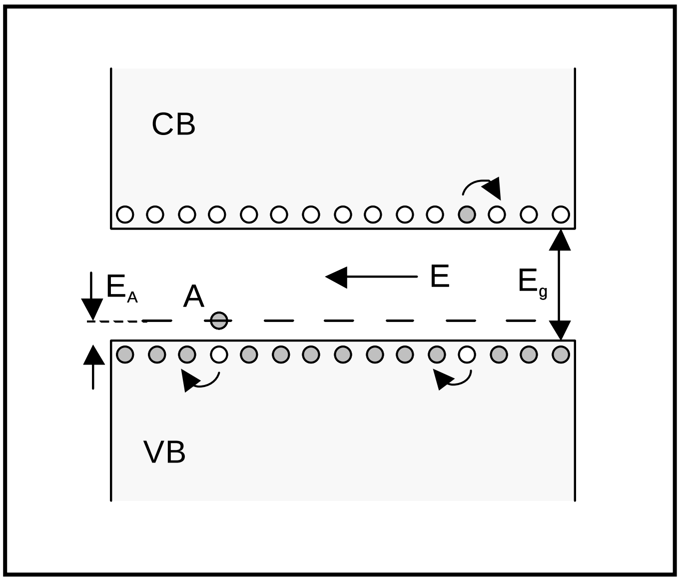

Fig. 2: Simplified diagram of extrinsic (left) and intrinsic conduction (right) under influence of an electric field E: Incorporating of dopants (acceptors A) into the crystal lattice creates positive charge carriers called holes in the valence band (VB). With increasing temperature the thermal energy of valence electrons increases allowing them to breach the energy gap E_g into the conduction band (CB) leaving a vacancy called hole in the VB.

Their activation energy E_A of about 0.01 eV is significantly smaller than the activation energy E_g (band gap) to generate electrons and holes by thermal activation (intrinsic charge carriers). At room temperatures in p-doped germanium the density of holes p_S can predominate the density of intrinsic charge carriers (p_E and n_E). In this case where the charge transport is predominately due to holes from the dopants ($n = n_E = p_E \approx 0$). The density of p_S can be determined by measuring the Hall voltage U_H as function of the current I. With equation (I) and (II) follows:

$$p_S = \frac{B}{e_0 \cdot d} \cdot \frac{I}{U_H} \qquad \text{(III)}$$

The mobility is a measure of the interaction between the charge carriers and the crystal lattice. The mobility is defined as (in case p-doped germanium it is the mobility μ_P of the holes created by the dopants, i.e. acceptors):

$$\mu_P = \frac{v_p}{E} \qquad \text{(IV)}$$

v_p: drift velocity

E: electric field due to the voltage drop

The electric filed E can be determined by the voltage drop U and the length w of the p-doped germanium strip:

$$E = \frac{U}{w} \qquad \text{(V)}$$

The drift velocity v_p can be determined from the equilibrium condition, where the Lorentz force compensates the electrical force which is due to the Hall field (Fig. 1)

$$e_0 \cdot v_d \cdot B = e_0 \cdot E_H \qquad \text{(VI)}$$

which can be expressed using the relation $E_H = b \cdot U_H$ as

$$v_d = \frac{U_H}{b \cdot B} \qquad \text{(VII)}$$

Substituting equation (V) and (VII) in equation (IV) the mobility μ_p of holes can be estimated at room temperatures as follows:

$$\mu_p = \frac{U_H \cdot w}{b \cdot B \cdot U} \qquad \text{(VIII)}$$

The current I in a semiconductor crystal is made up of both hole currents and electron currents (Fig. 1):

$$I = b \cdot d\,(n_p \cdot \mu_p + n_n \cdot \mu_n) \qquad \text{(IX)}$$

The carrier density depends on the dopant concentration and the temperature. Three different regions can be distinguished for p-doped germanium: At very low temperatures the excitation from electrons of the valence band into the acceptor levels is the only source of charge carriers. The density of holes p_S increases with temperature. It follows a region where the density p_S is independent of temperature as all acceptor levels are occupied (extrinsic conductivity). In this regime the charge transport due to intrinsic charge carriers can be neglected. A further increase in temperature leads to a direct thermal excitation of electrons from the valence band into the conduction band. The charge transport increases due to intrinsic conductivity and finally predominates (Fig. 2). These transition from pure extrinsic conduction to a predominately intrinsic conduction can be observed by measuring the Hall voltage U_H as function of the temperature.

To describe the Hall voltage as function of temperature U_H based on a simple theory equation (I) and (II) have to be extended in the following way:

It is assumed that the mobility of electrons and holes are different. Introducing the ratio of the mobility

$$k = \frac{\mu_n}{\mu_p} \qquad \text{(X)}$$

equation (II) can be rewritten as follows:

$$R_H = \frac{1}{e_0} \cdot \frac{p - n \cdot k^2}{(p + n \cdot k)^2} \qquad \text{(XI)}$$

For undoped semiconductors the temperature dependency of the charge carriers can be assumed as

$$n = n_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}}$$

$$p = p_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}} \qquad \text{(XII)}$$

$k_B = 1.36\ 10^{-23}$ J/K: Boltzmann constant

The product of the densities p and n is temperature dependent:

$$n \cdot p = n_E \cdot (p_E + p_S) = \eta^2 \qquad \text{(XIII)}$$

where the effective state density η is approximated as

$$\eta^2 = N_0 \cdot e^{-\frac{E_g}{k_B \cdot T}} \qquad \text{(XIV)}$$

In the extrinsic conductivity regime the density p_S of holes can be determined according equation (III). For the intrinsic charge carriers $p_E = n_E$ which leads to a quadratic equation for p_E with the solution:

$$p_E = -\frac{p_S}{2} + \sqrt{\frac{p_S^2}{4} + \eta^2} \qquad \text{(XV)}$$

With equations (XI) and (XV) together with the relations $p = p_E + p_S$ and $n = n_E$ the temperature dependency of Hall voltage U_H can be simulated. Using for E_g = 0.7 eV the result of experiment P7.2.1.5 as estimate value for the simulation only two unknown parameters N_0 and k are left.